Mittuniversitetet

MID SWEDEN UNIVERSITY

Summer course in Several Complex Variables at University of São Paulo

January-February 2026

Content of Summer Course in Several Complex Variables

Lecture 1 - $\mathbb{C}$ versus $\mathbb{C}^n$

The material for this lecture can be found in Chapter 0 in [K]. There are many differences between one and several variables. In this lecture we will go through some of these differences, for example domain of holomorphy, zeros of holomorphic functions, biholomorphic mappings and domain of convergence. These examples motivates the theory of several complex variables.

Lecture 2 - holomorphic functions and mappings on $\mathbb{C}^n$

In this lecture we extend the notion of holomorphicity to several variables. We will see that the Cauchy-Riemann's equations extends to $\mathbb{C}^n$. We will also see some other extensions from one variable theory, and study local properties of holomorphic functions and mappings. We will follow parts from chapter 1 and chapter 2 in [R].

Lecture 3 - The Riemann's mapping theorem doesn't extend to several variables

We continue to study holomorphic mappings, and the main result of this lecture is that the Riemann's mapping theorem from one variable doesn't extend to several variables. We will also study the zero set of holomorphic functions, i.e. analytic sets. We will follow [R] in this lecture.

Lecture 4 and 5 - Different kind of convexity

There are many notions of convexity. We will spend a couple of lectures introducing domain of holomorphy, convexity with respect to a class of functions and pseduoconvexity. We will follow both [R] and [K] in these lectures.

Lecture 6 and 7 - Complex structures and holomorphicity

Complex structures are the thing that separates the complex from the real. In these lectures we will see the background of the notion of complex structures and something called the complexification. We will connect the dots to holomorphicity for holomorphic mappings. We will follow Andreas' lecture notes and [R] in these lectures.

Lecture 8 and 9 - Integral formulas

In one variable complex analysis it is a classic fact that we can represent a value of function as an integral of itself, the so called Cauchy's Integral Formula. This fact is due to something called a reproducing kernel, the Cauchy Kernel. We will see in these lectures that there are other Integral formulas and kernels. We will follow both [R] and [K] in these lectures.

Lecture 10 and 11 - On the solution of the $\bar{\partial}$ equation

The most famous equation in complex analysis is the $\bar{\partial}$ equation. Ít says, given a function/differential form g on some domain/space, is it possible to find a function/differential form f such that $\bar{\partial} f = g$? The question depends heavily on the regularity of both the starting function/differential form g and the regularity of the domain/space where it lives. In these lectures we will address this huge problem, and see some results. The lectures will be part of [R] and [K].

Bibliography

[K] Steven G. Krantz: *Function Theory of Several Complex Variables*, AMS Chelsea Publishing 2000.

[R] R. Michael Range: *Holomorphic Functions and Integral Repre sentations in Several Complex Variables*, Graduate Texts in Mathematics, Springer-Verlag 1998.