Inequalities for orthonormal systems with applications: from stability of matter to quantum dynamics

Konstantin Merz (ETH Zürich)

Abstract: Inequalities play a central role in partial differential equations, harmonic analysis, spectral theory, and applied mathematics.

In this mini-course, we will see how inequalities connect to fundamental questions in quantum physics. In the first part, we review a selection of classical inequalities and explain how they lead to the stability of matter - a fundamental fact of daily life that most of us take for granted, but is not trivial to prove. The proof requires us to study inequalities for systems of functions. We will learn how constraints such as orthonormality yield much stronger inequalities than what one would obtain by merely applying the triangle inequality. In particular, the proof reveals that the ultimate reason for the stability of matter is the fermionic nature of electrons.

In the second part, we turn to dispersive estimates for the Schrödinger equation, extend them to orthonormal systems, and apply these inequalities to study the dynamics of many-particle quantum systems described by the Hartree equation.

References:

Lieb, Elliott H. and Seiringer, Robert. *The Stability of Matter in Quantum Mechanics.* Cambridge University Press, Cambridge, 2010.

Frank, Rupert L. *Lieb-Thirring inequalities and other functional inequalities for orthonormal systems.* International Congress of Mathematicians. EMS Press, Berlin, 2023.