Proposed course – Aris Daniilidis

Short Biography. Aris Daniilidis has an ample teaching experience having delivered regular courses in institutions of five countries (Austria, France, Greece, Spain, Chile). In particular, he taught at the Universities of Pau (1999-2001), Grenoble (2002), Tours (2008), Aegean (1996, 2001), Autonomous of Barcelona (2005-2013), University of Chile (2013-2021) and since 2021 he lectures at the TU Wien. He also delivered Lectures in Advanced Courses (spring, summer schools) and at the EMALCA School. His courses have been addressed to a variety of audience (engineers, university, advances schools) and to all possible levels (L1–L3, M1–M2, Doctorate level). Among the regular courses of the programme of the Department of Mathematical Engineering, Aris Daniilidis has taught Calculus of Various Variables (L2), Differential and Variational Calculus (L3), General Topology (L3), Functional Analysis (M1), Convex Analysis and Duality Theory (M1) and Advanced Topics in Functional and Variational Analysis (M2).

The proposed course for Advanced Master/PhD students and researchers is entitled:

- **General determination theory**

The course is conceived of total duration of 4 lectures.

Description of the course. The starting point of this course will be a recent result of convex analysis, namely that every smooth convex and bounded from below function f defined on a Hilbert space is determined, up to a constant, by the modulus of its derivative $\|\nabla f\|$. In other words, mere knowledge of the norm of the gradient function $\|\nabla f\|$ provides information also about its direction. We refer to this type of result, as *determination result*.

We shall first see that the above determination result extends to the nonsmooth case (via subdifferential theory), where the distance of the subdifferential to 0 plays the role of $\|Df\|$. Then we shall gradually move to the framework of metric spaces, omitting the linear structure, and replacing $\|Df\|$ by the De Giorgi metric slope. In this case, we shall see that a determination result can also be established for the class of bounded from below Lipschitz continuous functions, by adding information on the critical values. Finally, we shall consider more general situations (stochastic processes), by means of an abstract modulus of descent, encompassing both steepest and average descents) giving rise to a general determination theory. We shall also discuss analogies to the viscosity theory of PDEs (Eikonal equations) and to the w-KAM theory.

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