

Minicourse: Introduction to Lie algebras

Prof. Uirá Norberto Matos de Almeida

1 Resume

This minicourse aims to give a gentle introduction to the structure theory of Lie algebra with a focus on the structure of Semisimple Lie algebras. We present Lie's and Engel's theorems on Solvable and Nilpotent Lie algebras before working toward Cartan's classification theorem. The main requisite for this course is Linear Algebra, but a passing familiarity with other algebraic structures can help (groups and fields), thus the target public are math undergraduates going to their third year, but second years may also be able to keep up most of the time. More advanced students who never had contact with the subject may also have some interest in this course.

2 Detailed Course program

Bellow we detail the course program, organized in 2h classes.

2.1 Class 1

1. Introduction

- Symmetries and groups.
- Lie algebras as linear approximations of groups
- Some examples

2. Beginnings

- Definitions

- Solvable, Simple and Nilpotent algebras.

2.2 Class 2

1. Solvable Lie algebras and Lie theorem
2. Nilpotent Lie algebras and Engel's Theorem

2.3 Class 3

1. Killing Form and Cartan semisimplicity Criteria
2. Jordans canonical form (without proof).

2.4 Class 4

1. Weights and generalized weights spaces
2. Weight space decomposition
3. Cartan subalgebras
4. Sketch the proof of existence of Cartan Subalgebras for complex semisimple Lie algebras.
5. Cartan subalgebras are unique (without proof)

2.5 Class 5

1. Roots
2. Root systems and $SL(2, \mathbb{C})$
3. Abstract root systems and irreducibility

2.6 Class 6

1. Cartan matrix
2. Dynkin diagram
3. Weyl Group

3 References

Our main reference is Knapp's book ([1]) "Lie Groups beyond an introduction", but there are a few other known resources which could be helpful.

Referências

- [1] A. Knapp *Lie Groups beyond an introduction*. Progress in Mathematics, 140, Birkhäuser Basel, 2002.
- [2] S. Helgason *Differential Geometry, Lie Groups, and Symmetric Spaces*. Academic Press, 1978.
- [3] J. Humphreys *Introduction to Lie Algebras and Representation Theory*. Graduate Texts in Mathematics, 9, Springer-Verlag New York, 1978