Elliptic PDEs and measure theory: Old and new

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Abstract:
In this minicourse we develop an elliptic PDE theory adapted to study existence and properties of solutions in cases where little regularity is available from the data, normally measures. Specifically, we are interested in the Dirichlet problem

$$\begin{cases} -\Delta u + Vu = \mu & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

Our final goal is to prove that for every $p > 1$ and for every potential $V \in L^p$, any nonnegative function satisfying the equation $-\Delta v + Vv \geq 0$ in an open connected set of $\mathbb{R}^N$ is either identically zero or its level set $\{v = 0\}$ has zero $W^{2,p}$ capacity. This gives an affirmative answer to a conjecture of Bénilan and Brezis concerning a bridge between Trudinger's strong maximum principle for $p > \frac{N}{2}$ and Ancona's strong maximum principle for $p = 1$. The minicourse is based on a book in preparation (preliminary version available at `arxiv.org`) and on a recent work with L. Orsina (Sapienza — Università di Roma).

Program:

- Bénilan-Brezis conjecture;
- Strong maximum principle in the borderline cases: $V \in L^\infty$ and $V \in L^1$;
- Background on measures and capacities;
- Linear regularity theory involving L^1 and measure data;
- Weak formulation of maximum principles and Kato's inequality;
- Solutions with potentials $V \in L^p$;
- Existence of diffuse measures (Hahn-Banach theorem);
- Strong approximation of measures (Maz'ya's strong capacitary estimate);
- Strong maximum principle for potentials $V \in L^p$;
- Extensions and open directions.

Background:
Basic knowledge of measure theory, functional analysis and a first course on elliptic PDEs.