Some Dynamical Properties of Homogeneous Quadratic Systems

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In this talk we will consider the one-to-one correspondence between homogeneous quadratic systems of ODEs $x' = Q(x)$ and homogeneous quadratic discrete systems $x_{k+1} = Q(x_k)$ (DDS) and nonassociative commutative finite dimensional real algebras [1-3]. We will consider some well-known results [1,2] for homogeneous quadratic systems of ODEs and DDS arising from (to) the (system) associated algebra multiplication $*$, defined by $x * y = \frac{Q(x+y)-Q(x)-Q(y)}{2}$, as well as some original results [3,5,6]. The most important role in the theory of algebraic approach to homogeneous systems plays the existence of two special algebraic elements called idempotents $x * x = x$ and nilpotents (of rank two) $x * x = 0$; $x \in \mathbb{R}^n$.

Most of the theory holds true for $x \in \mathbb{R}^n$. Some special algebraic structure (like the existence of ideals and subalgebras) have some interesting influence on the dynamics in the corresponding continous/discrete dynamical system. The meaning of algebra isomorphism is equal in both cases and it represents the basis for the linear equivalence classification of homogeneous quadratic systems. However, some results on (non)chaotic dynamics for DDS are limited to $x \in \mathbb{R}^2$ [5]. We use the algebraic classification of 2D commutative real algebras [2] defined by: $\vec{e}_1 * \vec{e}_1 = a_1\vec{e}_1 + a_2\vec{e}_2$, $\vec{e}_1 * \vec{e}_2 = \vec{e}_2 * \vec{e}_1 = b_1\vec{e}_1 + b_2\vec{e}_2$, $\vec{e}_2 * \vec{e}_2 = c_1\vec{e}_1 + c_2\vec{e}_2$ (where $a_{1,2}, b_{1,2}, c_{1,2} \in \mathbb{R}$) in order to investigate the dynamics of (the corresponding) maps

$$\begin{array}{l} x_{k+1} = a_1 x_k^2 + 2b_1 x_k y_k + c_1 y_k^2 \\ y_{k+1} = a_2 x_k^2 + 2b_2 x_k y_k + c_2 y_k^2 \end{array} ; \quad a_{1,2}, b_{1,2}, c_{1,2} \in \mathbb{R} \tag{1}$$

There is no chaotic behavior in $\mathbb{R}^2$ in the continuous case, but on the other hand, it is well known that there is a chaotic behavior in some discrete cases (1) oc-

cur on the boundary of the set of all points with bounded forward orbits. The (non)chaotic dynamics of some maps of the form (1) will be presented.
Concerning the original results in continuous homogeneous quadratic systems, we will briefly consider the partial classification of commutative 3D algebras corresponding to

$$\begin{array}{l} x' = 2a_1xz + 2b_1yz + c_1z^2 \\ y' = 2a_2xz + 2b_2yz + c_2z^2 \\ z' = 2a_3xz + 2b_3yz + c_3z^2 \end{array} \; ; \quad a_{1,2,3}, b_{1,2,3}, c_{1,2,3} \in \mathbb{R} \tag{2}$$

from [6] and some results on stability of the (total degenerated) nonhyperbolic singularity of (2); c.f. [4]. Note that the obtained algebraic results can easily be generalized to any dimension. The generalization is a conjecture, which will also be presented in the report.

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Investigation of center manifolds of some three-dimensional systems and the isochronicity problem

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In this talk I will mostly present the results from [3], where a quadratic 3D system of ODEs

$$\begin{aligned}\dot{u} &= -v + au^2 + av^2 + cuw + dvw \\ \dot{v} &= u + bu^2 + bv^2 + euw + fvw \\ \dot{w} &= -w + Su^2 + Sv^2 + Tuw + Uvw\end{aligned} \tag{3}$$

with real coefficients a, b, c, d, e, f, S, T and U was investigated. System (3) was studied already in [1], and further in [2,3], where planar polynomial systems of ODEs appearing on the center manifold of (3) were studied. Using the solutions of the center-focus problem from [1], confirmed in [3] by the so called modular approach [4], we present the investigation of four (at most three parameter-) families of (at most third degree) polynomial systems corresponding to the center varieties of (3). Thus, all systems under consideration correspond to a center manifold filled with closed trajectories (corresponding to periodic solutions of (3)).

In particular, in the talk I shall present the criteria on the coefficients of the system to distinguish between the cases of isochronous and non-isochronous oscillations, considered in [2,3]. Bifurcations of critical periods of the system will be presented as well.

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