Invariant polynomials: applications in qualitative study of differential systems

Abstract of the course

Course objectives

are to present: some basic notions of invariant polynomials with respect to the subgroups of the group of affine transformations; the methods of their construction and the applications of these polynomials to the problems of integrability and classification of some families of autonomous polynomial systems of ODEs.

Course description

1^{st} *lecture.* Introduction. Tensor notation of differential systems. GL-invariants of linear systems. Concept of a polynomial basis of invariants.

2^{nd} *lecture.* Operations on tensors. The Fundamental Theorem. The basis of GL-invariants for linear systems. Construction of affine invariant polynomials. The structure of the set of GL-invariant polynomials. T-comitants, CT-comitants. Gram's Theorem.

3^{rd} *lecture.* Affine invariant polynomials which are responsible for the number and multiplicities of singularities (finite and infinite). The defining triangle and its geometrical meaning.

4^{th} *lecture.* Rational integrability. First integrals in invariant form. Polynomial integrability. The complete classification of polynomial integrable quadratic systems.

5^{th} *lecture.* Invariant polynomials which are responsible for the existence of invariant lines. The global classifications of quadratic and cubic systems with maximal number of invariant lines.

6^{th} *lecture.* Weak singularities (foci, centers, saddles) of differential systems. Trace polynomials. The complete classifications of weak singularities for the family of quadratic systems.

References:

1. P.J. Olver. *Classical Invariant Theory*, (London Mathematical Society student texts: **44**), Cambridge University Press, 1999.

2. K. S. Sibirsky. (*Introduction to the algebraic theory of invariants of differential equations.* Engl. transl. Manchester Univ. Press, Manchester, 1988.

3. G.B. Gurevich *Foundations of the Theory of Algebraic Invariants*, P. Noordholph Ltd., Groningen, Holland,1964.

4. D. Hilbert. *Theory of algebraic invariants*, Cambridge University Press, 1993.

5. J.C. Artes, J. Llibre, N. Vulpe. *Quadratic systems with a polynomial first integral: a complete classification in the coefficient space.* J. Differential Equations. 246 (2009), 3535-3558.

6. N. Vulpe, *Characterization of the finite weak singularities of quadratic systems via invariant theory.* Nonlinear Analysis. Theory, Methods and Applications, 74 (2011), No. 4, 6553–6582.